Bold letters means matrix or vectors: $\boldsymbol{A}$

Bold letters with subscripts means the i th row and j th column block matrix in matrix $\boldsymbol{A}$: $\boldsymbol{A}_{ij}$

The k th row and l th column element in i th row and j th column block matrix $\boldsymbol{A}_{ij}$ in matrix $\boldsymbol{A}$: $(A_{ij})_{kl}$

0.1. 0.1 Eq(14)

$$
\begin{aligned}
U(\xi,\eta,\zeta) = \sum_{i=1}^{I_1}\sum_{j=1}^{J_1}\sum_{k=1}^{K_1} B^1_{ijk}P_i(\xi)P_j(\eta)P_k(\zeta),\\
V(\xi,\eta,\zeta) = \sum_{i=1}^{I_2}\sum_{j=1}^{J_2}\sum_{k=1}^{K_3} B^2_{ijk}P_i(\xi)P_j(\eta)P_k(\zeta),\\
W(\xi,\eta,\zeta) = \sum_{i=1}^{I_3}\sum_{j=1}^{J_3}\sum_{k=1}^{K_3} B^3_{ijk}P_i(\xi)P_j(\eta)P_k(\zeta).
\end{aligned}
\tag{1}
$$

We use the following relation
$(i,j,k) = (1,1,1),(1,1,2),\ldots,(1,1,K),(1,2,1),\ldots,(1,2,K),\ldots,(1,J,K),\ldots,(I,J,K) \to p = 1,2,3\ldots,IJK$
to transform the counting indices (i,j,k) to a new index p. Then, we collect the coefficients $B^m_{ijk}, m = 1,2,3$ into new vectors $\boldsymbol{A}_m$ where the pth element $(\boldsymbol{A}_m)_p = B^m_{ijk}$. And we collect all the coefficients into a big vector which is $\boldsymbol{A} = (\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3)$.Therefore, we have

$$
\begin{aligned}
U(\xi,\eta,\zeta) = \sum_{i=1}^{I_1}\sum_{j=1}^{J_1}\sum_{k=1}^{K_1} (A_1)_p P_i(\xi)P_j(\eta)P_k(\zeta),\\
V(\xi,\eta,\zeta) = \sum_{i=1}^{I_2}\sum_{j=1}^{J_2}\sum_{k=1}^{K_3} (A_2)_p P_i(\xi)P_j(\eta)P_k(\zeta),\\
W(\xi,\eta,\zeta) = \sum_{i=1}^{I_3}\sum_{j=1}^{J_3}\sum_{k=1}^{K_3} (A_3)_p P_i(\xi)P_j(\eta)P_k(\zeta).
\end{aligned}
\tag{2}
$$

##Eq 16

$$
\Pi = V_{\max} - T_{\max} \to L = V_{\max} - T_{\max} \tag{3}
$$

0.1. 0.2 Eq 17

$$
\frac{\partial L}{\partial B^1_{ijk}} = 0, \frac{\partial L}{\partial B^2_{ijk}} = 0, \frac{\partial L}{\partial B^3_{ijk}} = 0. \tag{4}
$$

By using the index transform, we have

$$
\frac{\partial L}{\partial (A_1)_p} = 0, \frac{\partial L}{\partial (A_2)_p} = 0, \frac{\partial L}{\partial (A_3)_p} = 0. \tag{5}
$$

0.1. 0.3 Eq21

$$
\begin{bmatrix} \boldsymbol{\Pi}_{11} & \boldsymbol{\Pi}_{12} & \boldsymbol{\Pi}_{13} \\ \boldsymbol{\Pi}_{21} & \boldsymbol{\Pi}_{22} & \boldsymbol{\Pi}_{23} \\ \boldsymbol{\Pi}_{31} & \boldsymbol{\Pi}_{32} & \boldsymbol{\Pi}_{33} \end{bmatrix} \begin{bmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \\ \boldsymbol{A}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}. \tag{6}
$$

where

$$
(\Pi_{ij})_{pq} = \frac{\partial^2 L}{\partial (A_i)_p \partial (A_j)_q} \tag{7}
$$

In fact,

$$
\boldsymbol{\Pi} = \boldsymbol{K} - \omega^2 \boldsymbol{M} \tag{8}
$$

and

$$(K_{ij})_{pq} = \frac{\partial^2 V_{\max}}{\partial(A_i)_p \partial(A_j)_q} \tag{9}$$

$$\omega^2 (M_{ij})_{pq} = \frac{\partial^2 T_{\max}}{\partial(A_i)_p \partial(A_j)_q} \tag{10}$$

$$\begin{aligned}
\Pi_{11} = \frac{\partial L}{\partial(A_1)_p} =& \frac{Ec}{2\lambda(1+v)} \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \left\{ \frac{v}{1-2v}(\bar{\varepsilon}_{\xi\xi} + \bar{\varepsilon}_{\eta\eta} \right. \\
& + \bar{\varepsilon}_{\zeta\zeta})\lambda P_l(\xi)\frac{\partial P_m(\eta)}{\partial(\eta)} P_n(\zeta) + \lambda \bar{\varepsilon}_{\eta\eta} P_l(\xi) \frac{\partial P_m(\eta)}{\partial(\eta)} P_n(\zeta) \\
& + \frac{1}{2}\left[\bar{\varepsilon}_{\xi\eta} \frac{\partial P_l(\xi)}{\partial(\xi)} P_m(\eta) P_n(\zeta) \right. \\
& \left.\left. + \frac{\lambda}{\gamma} \bar{\varepsilon}_{\eta\zeta} P_l(\xi) P_m(\eta) \frac{\partial P_n(\zeta)}{\partial(\zeta)} \right] \right\} d\xi d\eta d\zeta \\
& + \frac{\bar{E}c}{2\lambda(1+\bar{v})} \int_{-1}^{1}\int_{-1}^{1}\int_{1}^{1+\frac{2\bar{c}}{c}} \left\{ \frac{\bar{v}}{1-2\bar{v}}(\bar{\varepsilon}_{\xi\xi} + \bar{\varepsilon}_{\eta\eta} + \bar{\varepsilon}_{\zeta\zeta}) \right. \\
& \cdot \lambda P_l(\xi)\frac{\partial P_m(\eta)}{\partial(\eta)} P_n(\zeta) + \lambda \bar{\varepsilon}_{\eta\eta} P_l(\xi) \frac{\partial P_m(\eta)}{\partial(\eta)} P_n(\zeta) \\
& + \frac{1}{2}\left[\bar{\varepsilon}_{\xi\eta} \frac{\partial P_l(\xi)}{\partial(\xi)} P_m(\eta) P_n(\zeta) \right. \\
& \left.\left. + \frac{\lambda}{\gamma} \bar{\varepsilon}_{\eta\zeta} P_l(\xi) P_m(\eta) \frac{\partial P_n(\zeta)}{\partial(\zeta)} \right] \right\} d\xi d\eta d\zeta \\
& - \left\{ \frac{\rho}{8} abc\omega^2 \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} V\left[P_l(\xi) P_m(\eta) P_n(\zeta) \right] d\xi d\eta d\zeta \right. \\
& \left. + \frac{\bar{\rho}}{8} abc\omega^2 \int_{-1}^{1}\int_{-1}^{1}\int_{1}^{1+\frac{2\bar{c}}{c}} V\left[P_l(\xi) P_m(\eta) P_n(\zeta) \right] d\xi d\eta d\zeta \right\}
\end{aligned} \tag{11}$$